Metal-insulator transition in spatially-correlated random magnetic field system

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We reexamine the problem of delocalization of two-dimensional electrons in the presence of random magnetic field. By introducing *spatial correlations* among random fluxes, a well-defined metal-insulator transition characterized by a *two-branch scaling* of conductance has been demonstrated numerically. Critical conductance is found non-universal with a value around e^2/h. Interesting connections of this system with the recently observed $B = 0$ two-dimensional metallic phase (Kravchenko *et al.*, Phys. Rev. B **50**, 8039 (1994)) are also discussed.

71.30.+h, 73.20.Fz, 73.20.Jc

Whether two-dimensional (2D) electrons can become delocalized in the presence of random magnetic field (RMF) is still controversial. This is a very important issue related to many interesting systems, like half-filled quantum Hall effect (QHE) [1,2], gauge-field description [3] of high-T_c superconductor and so on. By using the standard transfer-matrix method [4], a number of numerical calculations [5–7] have been performed for a non-interacting 2D electron system subject to *spatially-uncorrelated* RMF. The results indicate that electrons are always localized near the band edge, while there is a dramatic enhancement of localization length as one moves towards the band center. However, the interpretation of the latter is rather conflicting, ranging from that all states are still localized [5,7] with an extremely large localization length close to the band center to the existence of a critical region [6] with divergent localization length. Even if a critical region characterized by wavefunctions with fractional dimensionality [8] could exist here, a metallic phase seems being ruled out by those numerical calculations since a two-branch scaling as a hallmark for metal-insulator transition (MIT) has never been found. Analytically, while the study based on a perturbative nonlinear sigma model approach pointed [9] to the localization of all states, the existence of extended states was shown [10] possible in the presence of a long-range logarithmic interaction of the topological density (due to fluctuating Hall conductance [11]), which is supported by direct numerical calculations [12,13] of topological Chern number for the case of spatially-uncorrelated RMF with *reduced* field strength.

In contrast to spatially-uncorrelated RMF, however, magnetic flux fluctuations in realistic systems [1–3] may be much more smooth with finite-range spatial correlations. Such a smoothness can significantly reduce the random scattering effects while still retain the delocalization effect [10–13] introduced by magnetic fluxes. In this paper, we demonstrate numerically for the first time the existence of MIT which is characterized by a two-branch scaling of conductance in the presence of spatially-correlated RMF. The critical conductance itself is non-universal, with its value around e^2/h which generally increases as the Fermi energy shifts towards the band center. With much reduced error bar, the present numerical algorithm is also applied to an uncorrelated (white noise limit) RMF case and the results unambiguously show that all states are localized without a critical region at *strong* strength of RMF. Possible connections of the present RMF system to the zero-magnetic-field (B=0) 2D metal [14] are also discussed at the end of the paper.

We consider a tight-binding lattice model of noninteracting electrons under RMF. The Hamiltonian is defined as follows:

$$H = -\sum_{<ij>} e^{ia_{ij}} c_i^+ c_j + \sum_i w_i c_i^+ c_i \qquad (1)$$

Here c_i^+ is a fermionic creation operator, and $< ij >$ refers to two nearest neighboring sites. w_i is an uncorrelated random potential (white noise limit) with strength $|w_i| \leq W$. A magnetic flux per plaquette is given as $\phi(k) = \sum_{\Box} a_{ij}$, where the summation runs over four links around a plaquette labeled by k. We are interested in the case where $\phi(k)$ at different k's is correlated which can be generated in the following way:

$$\phi(k) = \frac{h_0}{\lambda_f^2/4} \sum_i f_i e^{-|R_k - R_i|^2/\lambda_f^2} \qquad (2)$$

where R_k (R_i) denotes the spatial position of a given plaquette $k(i)$. h_0 and λ_f are the characteristic strength and correlation length scale of RMF, respectively. f_i is a random number distributing uniformly between (-1,+1).

We employ the following numerical algorithm to calculate the longitudinal conductance G_{xx}. Based on the Landauer formula, G_{xx} for a square sample $\mathcal{N} = L \times L$ can be determined as a summation over contributions from all the Lyapunov exponents of the Hermitian transfer matrix product T^+T [4,15]. To reduce the boundary effect of a finite-size system, we connect M different square samples together to form a very long stripe along x direction [of size $L \times (LM)$]. Typically M is chosen to be larger than 5000 even for the largest sample size ($L = 200$) in this work. In this way, the statistical error

bar is significantly reduced in our results (about 1.5%). In most of earlier numerical calculations, finite-size localization length has been computed where the statistical fluctuation is usually quite big (especially near the band center) as compared to a direct calculation of the finite-size longitudinal conductance in the present algorithm.

As a test, we have first re-studied the case in which the flux $\phi(k)$ is randomly distributed between $-\pi$ to π without spatial correlations – the situation investigated previously [5–7] as mentioned at the beginning of the paper. We find that G_{xx} monotonically decreases with the sample size L at all strengths of the on-site disorders: from $W = 0$ to $W = 4$, and is extrapolated to zero at large sample-size limit as shown in Fig. 1 at a fixed Fermi energy $E_f = -1$. In the insert of Fig. 1, G_{xx} is shown as a function of the disorder strength W at different sample sizes: $L = 24$, 80, and 200, which shows that even at $W = 0$ the conductance monotonically decreases with the increase of L, indicating that the dominant role of the random flux here is similar to the random potential in causing localization of electrons. The one-parameter scaling of G_{xx} can be obtained by choosing a scaling variable ξ at each random potential W. As plotted in Fig. 2, all data can be then collapsed onto a single curve of L/ξ, in which ξ is given in the insert of Fig. 2. Clearly ξ is always finite although it becomes extremely large at weak disorder limit. This is consistent with the conclusion [5,7] that electrons are all localized and excludes the possibility of a critical region [6] as the error bar in our calculation is much less than the variation of the conductance itself. Notice that in weak-disorder limit ξ may no longer be interpreted as localization length [7] which characterizes an exponential decay of conductance with sample size at strong localized region.

Now let us focus on RMF with smooth spatial correlations as defined in (2). With the correlation length $\lambda_f = 5.0$ (the lattice constant as the unit) and flux strength $h_0 = 1$, G_{xx} as a function of disorder strength W is computed at a given Fermi energy $E_f = -1$ as shown in Fig. 3. Curves at different sample sizes ($L = 16 - 200$) all cross at a fixed-point $W = W_c$, which is independent of lattice size L within the statistical error bars. It is qualitatively different from the behavior of G_{xx} in spatially-uncorrelated RMF case discussed above. At $W > W_c$, G_{xx} continuously decreases with the increase of the sample size, which can be extrapolated to zero at large L limit, corresponding to insulating phase. On the other hand, at $W < W_c$, G_{xx} monotonically increases with lattice sizes like a typical metallic behavior. The insert of Fig. 3 shows the critical conductance G_c (corresponding to $W = W_c$) at different Fermi energies and h_0's. The data of G_{xx} in Fig. 3 can be collapsed onto a two-branch curve as a function of scaling variable L/ξ as shown in Fig. 4 for $W > W_c$ and $W < W_c$, respectively. The insert of Fig. 4 shows the scaling variable ξ vs. W which diverges at the critical point W_c. In the metallic phase at $W < W_c$, G_{xx} can be approximately fitted by the following form: $G_{xx} = G_s - c_0 * exp(-L/\xi_0)$. Here G_s is the saturated conductance at $L \to \infty$, which is non-universal and depends on the disorder strength W as well as the correlation length λ_f of random fluxes.

The introduction of spatial correlations in random fluxes is crucial for such a metal-insulator transition. We also found a well-defined MIT at an even shorter correlation length: $\lambda_f = 2.0$. But the larger λ_f is, the stronger the metallic behavior becomes with a larger saturated conductance. The previously discussed RMF in white noise limit may only belong to a very special case in which the localization effect of strong randomness of fluxes overwrites the delocalization effect of the same fluxes. We would like to point out that even in such an uncorrelated random flux case, the delocalization may be still enhanced if one reduces the *strength* of RMF. Earlier topological Chern number calculations [12] clearly indicates a delocalization transition as the maximum strength of $\phi(k)$ is reduced to around $\pi/2$. We have computed the conductance in this case using the present method at much larger sample sizes and indeed found a slight increase of the conductance with sample size at $W < W_c$, which is opposite to strong random flux limit where conductance always decreases with the increase of sample size (Fig. 1), although a two-branch scaling curves here is not as clear-cut as in the spatially-correlated RMF case shown in Fig. 4.

As mentioned above, the critical conductance G_c varies from $0.5e^2/h$ to around $2e^2/h$ as the Fermi energy shifts from the band edge towards band center (the insert of Fig. 3). It is interesting to note that G_c obtained here is in the same range as the experimental data found in recent $B = 0$ 2D MIT system [14]. In the following, we would like to point out a possible deeper connection between the two systems.

In a recent experiment [16] in p-type GaAs/AlGaAs heterostructure, the evolution of delocalized states was studied continuously from the QHE regime at strong magnetic field to zero field limit where the $B = 0$ MIT is recovered. The authors found that the critical density of the lowest extended level in QHE regime flattens out, instead of floating up towards infinity, as magnetic field is reduced and can be extrapolated to the critical density of $B = 0$ MIT in such a material. Similar result has been also observed in Si-MOSFET samples [17,18]. At first sight, it is tempting to think that the lowest extended level of QHE somehow survives at $B = 0$, but physically it does not make much sense because QHE extended states carry quantized Hall conductance known as Chern number whereas at $B = 0$ the total Hall conductance must be zero without time-reversal symmetry-breaking. In fact, experiments indicated [17] that before B vanishes, extended levels of the QHE may already merge with a different kind of extended level (called QHE/Insulator boundary in Ref, [17]) which carries an

opposite sign of Hall conductance. Theoretically, it has been previously found [19] that QHE extended states indeed can be mixed with some boundary extended level moving down from high-energy side at strong disorder or weak magnetic field limit which carries negative Chern number in a lattice model. When those extended states with different signs of Chern numbers mix together at weak magnetic field limit, there could be two consequences: one is that no states will eventually carry nonzero Chern number due to the cancellation such that all of them become localized. This is what happens in non-interacting system [19]; The second possibility is that individual states may still carry nonzero Chern numbers and form a delocalized *region* even though the *average* Hall conductance still vanishes at $B = 0$. Such a system is then physically related to the RMF system where the delocalization mechanism is also due to the fluctuating Hall conductance [10–13]. Below we give a heuristic argument how a strong Coulomb interaction may lead to such a realization.

At strong Coulomb interaction with $r_s \gg 1$ (here r_s is the ratio of the strength of the Coulomb interaction over the Fermi energy [14]), the 2D electron state is very close to a Wigner glass phase where the low-lying spin degrees of freedom may be described by an effective spin Hamiltonian H_s given in Ref. [20]. The low-lying charge degrees of freedom may be regarded as "defects" which can hop on the "lattice" governed by a generalized $t - J$ like model [20,21]. Based on many studies on the $t - J$ model in high-T_c problem, especially the gauge-field description [3], charge carriers moving on a magnetic spin background can generally acquire fictitious fluxes. Such kind of fluxes usually can be treated as random magnetic fields with some finite-range spatial correlations. According to the numerical results presented above, such a system indeed can have a MIT at $B = 0$. Of course, further model study is needed in order to fully explore this connection which is beyond the scope of the present paper.

In conclusion, we have numerically demonstrated the existence of a metal-insulator transition characterized by *a two-branch scaling* for 2D electrons in the presence of *spatially-correlated* random magnetic fields. In contrast to usual three-dimensional metal where the conductance scales to infinity, this 2D metal has a saturated non-universal conductance. The range of the critical conductance is very similar to that found in $B = 0$ 2D metal-insulator transition. We briefly discussed a possible connection between a 2D interacting electron system at $r_s \gg 1$ and the spatially-correlated random-magnetic-field problem based on both experimental and theoretical considerations.

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Fig. 1 The evolution of conductance G_{xx} (in units of e^2/h) with sample width L at differnent disorder strength W's for spatially-uncorrelated RMF case. The insert: G_{xx} as a function of W at different L's. Fermi energy is fixed at $E_f = -1$.

Fig. 2. The data of G_{xx} at different L's and W's all collapse onto a scaling curve as a function of L/ξ. The insert: ξ versus W.

Fig. 3 G_{xx} versus W at different sample sizes ($L = 16(\bullet), 24, 32, 48, 64, 80, 120, 200$). W_c is the critical disorder. Fermi energy is chosen at $E_f = -1$. The insert: critical conductance G_c as a function of Fermi energy E_f.

Fig. 4. Two branch-scaling curve of G_{xx} as a single function of L/ξ for different L's and W's. The insert: ξ versus W.

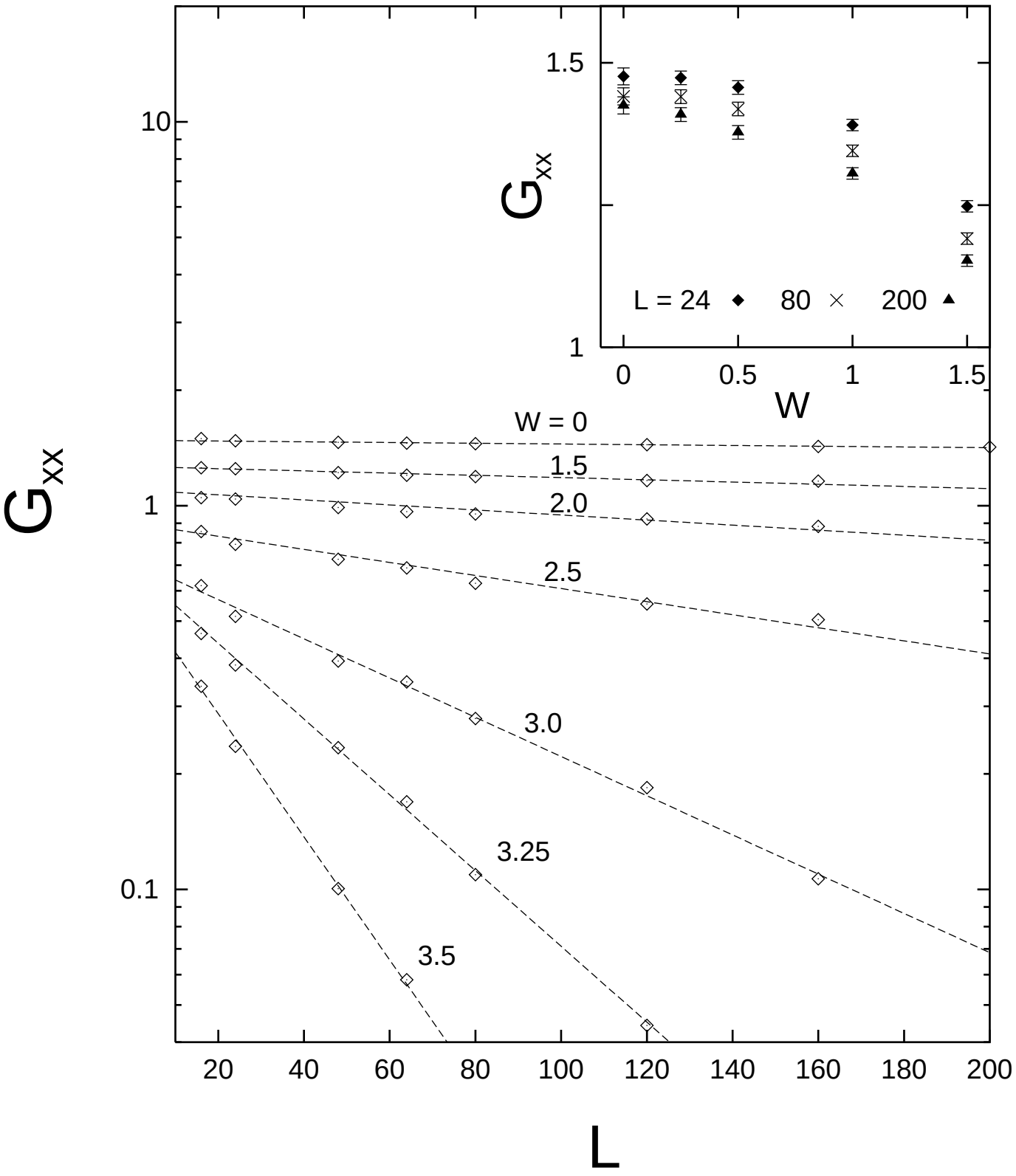
10
1
0.1
G_{xx}
W = 0
1.5
2.0
2.5
3.0
3.25
3.5
20
40
60
80
100
120
140
160
180
200
L
1.5
1
G_{xx}
L = 24
80
200
0
0.5
1
1.5
W

Fig. 1

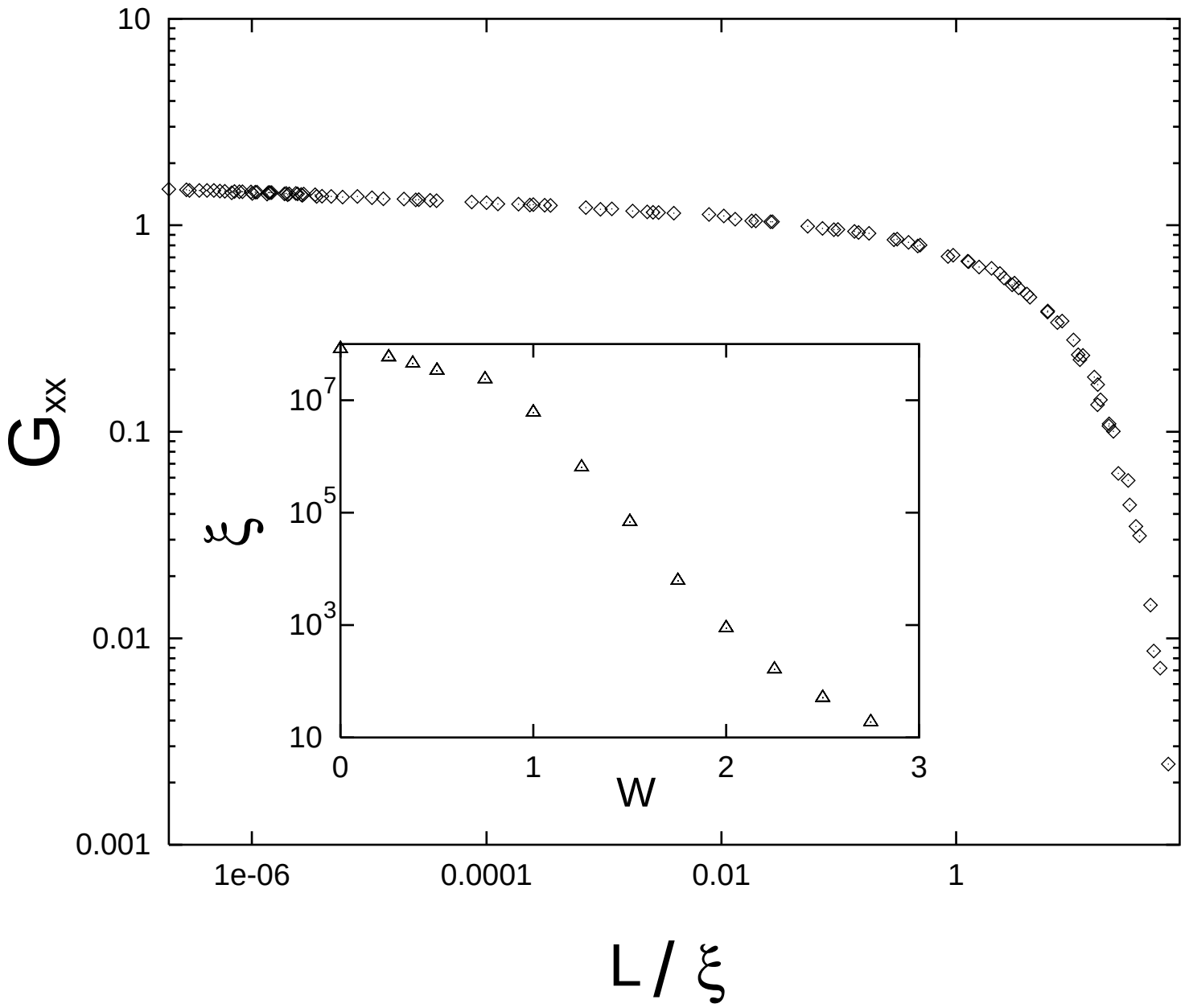

Fig. 2

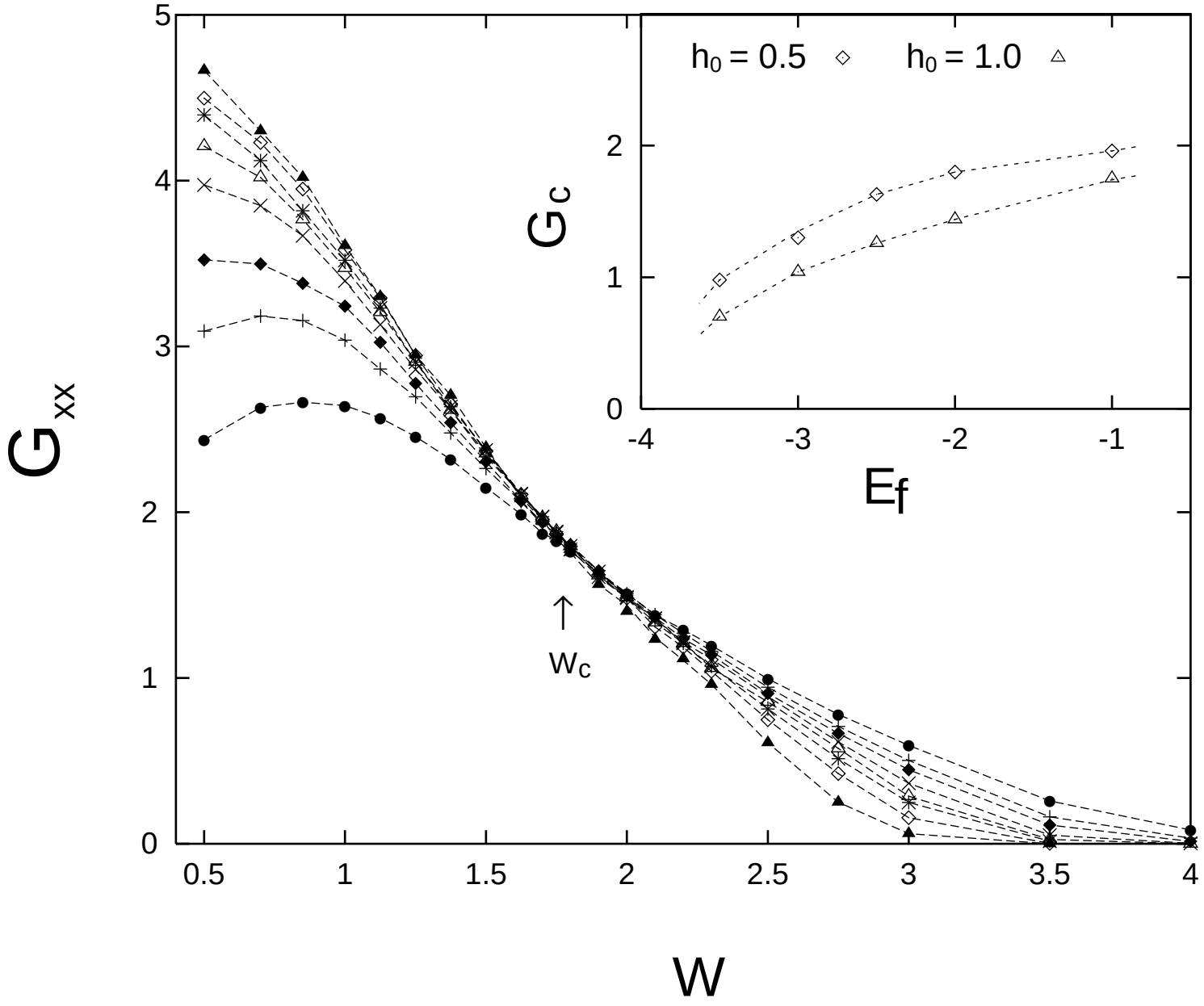

Fig. 3

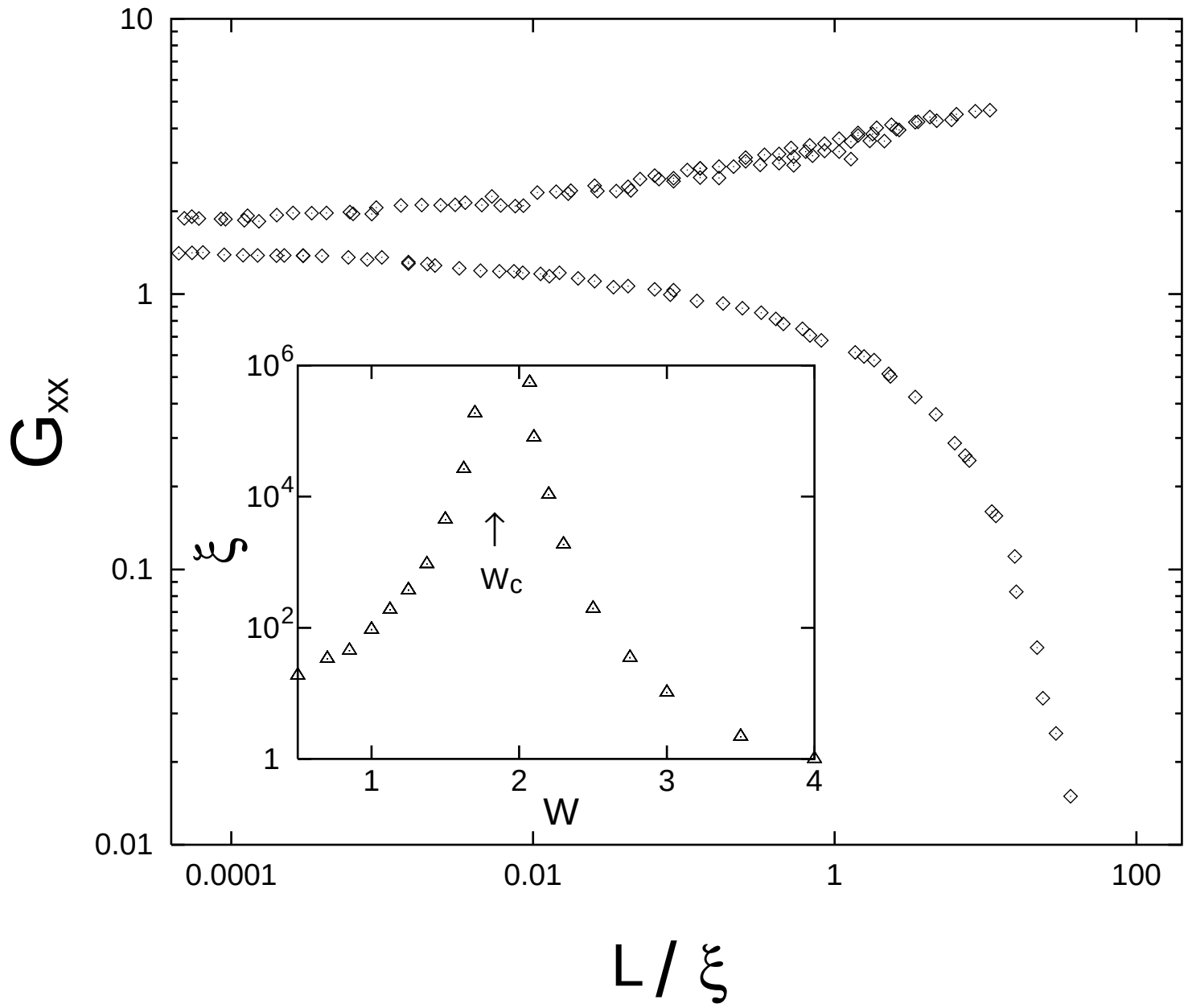

Fig. 4